Theoretical frameworks for financial literacy

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This session

The standard lifecycle consumption model ignores the role of financial literacy

In this session we discuss how one can incorporate financial literacy in an intertemporal consumption model.

Main references:

- Lusardi, A., Michaud, P. C., & Mitchell, O. S. (2017). Optimal financial knowledge and wealth inequality. Journal of political Economy, 125(2), 431-477 (see reading list).
- Jappelli, Tullio, and Mario Padula. 2013. Investment in Financial Knowledge and Saving Decisions. Journal of Banking and Finance 37(8): 2779–92.

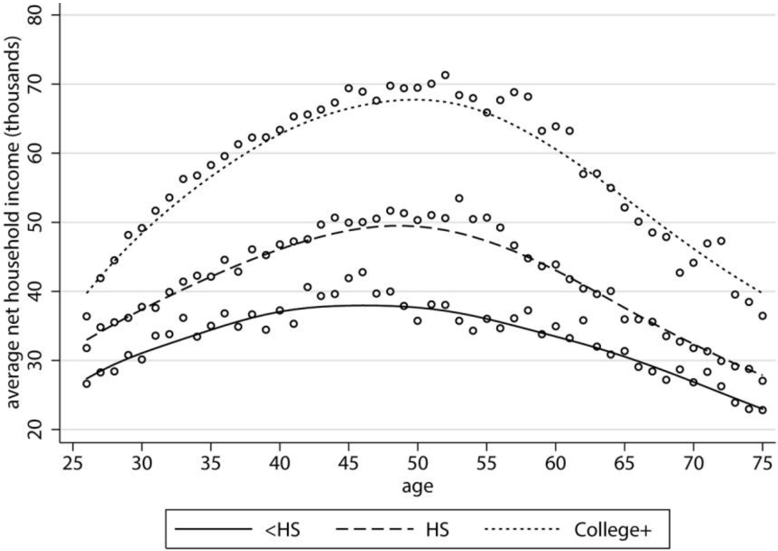
Motivation

- Americans are increasingly being asked to manage their own financial well-being.
- At the same time, financial markets have become more complex
- In this new environment, investments in financial knowledge can have important consequences for retirement wealth.
- When the decision to invest in financial literacy alters life cycle wealth profiles, it will help explain wealth inequality.

How to incorporate financial knowledge into standard life cycle models?

- Financial knowledge itself should be modeled as an endogenous choice variable akin to human capital investment.
- Three different models are presented to answer two research questions:
  1. What forces shape financial knowledge accumulation over the life cycle?
  2. How much wealth inequality can be attributable to resulting differences in financial knowledge?
- The three model are (in increasing order of complexity)
  1. Two period model under certainty (Jappelli & Padula, 2013; Lusardi et al., 2017)
  2. Multiperiod model under certainty (Jappelli & Padula, 2013)
  3. A stochastic life cycle model featuring uncertainty in income, capital market returns, and medical expenditures. It also incorporate an endogenous knowledge accumulation process and a sophisticated saving technology.

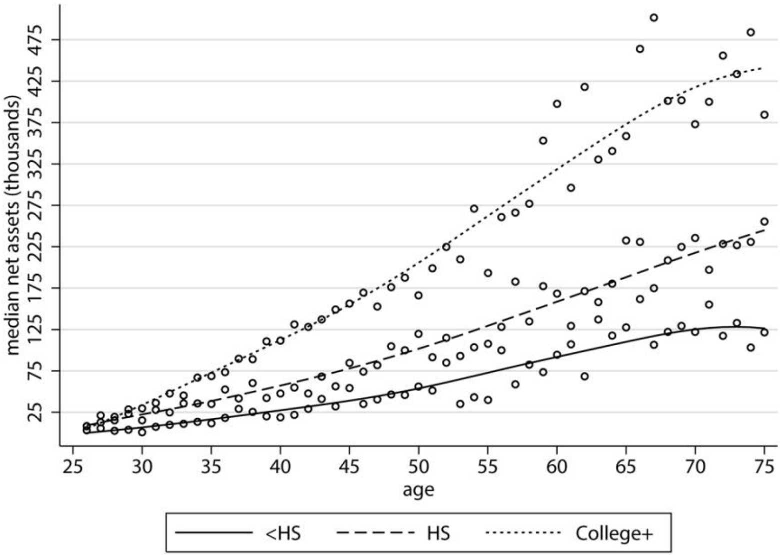
Life cycle household income profiles by educational level



Comments on income profiles by educational level

- Household income excludes income from capital
- After retirement, income falls because Social Security and pension benefit amounts are generally less than labor earnings.
- old-age benefit replacement rates are higher for the least educated because of the progressivity of public safety net programs
- Net household income declines somewhat, in part because of changes in household composition (e.g., loss of a spouse).

Life cycle wealth profiles by educational level



Comments on wealth profiles by educational level

- net wealth=bank accounts+stocks+IRAs+mutual funds, bonds+net real estate-debt.
- PSID does not have data on 401(k) plan balances however, included in income measure!
- striking differences by educational attainment
  - Median wealth college+: \$375,000
  - <HS: \$125,000

Conclusions from descriptive analysis (1)

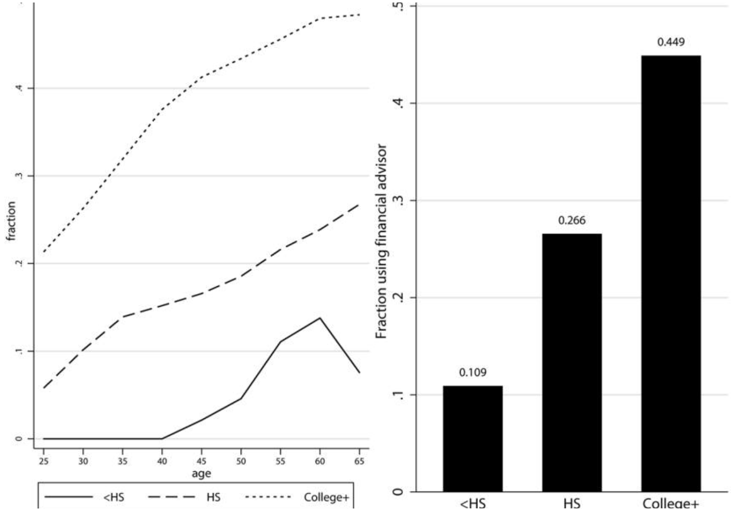
- Key prediction of a simple Life cycle model: the life cycle path of assets normalized by lifetime income should be the same across groups.
- Yet the data reveal non-proportional wealth-to-income profiles, implying that the simple life cycle model cannot explain observed wealth heterogeneity.
- More sophisticated models allow for
  1. Precautionary saving
  2. Preferences
  3. Demographics
  4. Differential mortality
  5. Consumption floor (means-tested and redistributive transfer programs such as Social Security, and Medicaid.
     Note: consumption floors cannot explain wealth inequality in the upper half of the income distribution

Differences in Sophisticated Financial Products by education

TABLE 1
Life Cycle Participation and Share of Portfolio in Sophisticated Financial Products (Stocks and IRAs) by Educational Attainment in the PSID: 2001–5

By Age	< High School	High School	College+	Total
	Participation Rate			
25–35	.158	.177	.541	.369
35–45	.173	.331	.623	.477
45–55	.193	.415	.651	.54
55–65	.28	.522	.752	.635
65–75	.421	.586	.732	.622
Total	.243	.39	.652	.52

Observed financial knowledge and use of financial advice



Comments on left panel figure 3

1. Financial knowledge differs considerably across education groups.
2. Financial knowledge increases over the life cycle
3. Observed differences in financial knowledge widen with age.
4. There is some evidence that financial knowledge eventually decreases past middle age

Comments on right panel figure 3

1. Few people use advisers, particularly among the least educated.
2. Advice is probably a complement to, rather than a substitute for, financial knowledge acquisition.

2-period model under certainty (Jappelli & Padula, 2013)

Assumptions

- Period 1: working life; period 2: retirement
- Only income in period 1 ($y_1 = y$), no income in period 2 ($y_2 = 0$)
  Due to this assumption, borrowing will never occur!
- Φ_t: stock of fin. knowledge at beginning of period t, $t = 1, 2$
- $\Phi_2 = (1 - \delta)\Phi_1 + \phi$
- $R(\Phi_2) = \Phi_2^\alpha$, $0 < \alpha < 1$ ($\alpha :=$ return to financial literacy)
- $p :=$ relative cost of literacy in terms of the consumption good.
- $s :=$ saving in period 1; $a = Rs$: wealth at the beginning of period 2.

How can financial literacy affect wealth? Alternative channels

- Expectations
- Preferences
- Underestimation of compounding effects
- FL may help people to avoid high transaction fees (c_d).
- Mazzonna and Peracchi (2012): human capital model cognitive abilities increase people's earnings before retirement.

Two period model Jappelli & Padula (2013)

$$\max_{s,\Phi_2} \Psi(s,\Phi_2) = \ln(\underbrace{y - p(\Phi_2 - (1-\delta)\Phi_1) - s}_{c_1}) + \beta \ln(\underbrace{\Phi_2^{\alpha} s}_{c_2})$$

$$\frac{\partial \Psi}{\partial s} = 0 \;\Leftrightarrow\; \frac{1}{(y - \pi\phi - s)} = \frac{\beta}{s} \;\Leftrightarrow\; s = \frac{\beta(y - p\phi)}{1+\beta}$$

$$\frac{\partial \Psi}{\partial \Phi_2} = 0 \;\Leftrightarrow\; \frac{p}{y - p\phi - s} = \frac{\alpha\beta}{\Phi_2} \;\Leftrightarrow\; p = \frac{\alpha\beta(y - p\phi)}{(\Phi_1(1-\delta) + \phi)(1+\beta)}$$

So, we can obtain the closed form solutions for ϕ and Φ_2:

$$\phi = \frac{\alpha\beta y/p - \Phi_1(1-\delta)(1+\beta)}{(1+\beta+\alpha\beta)} \tag{1}$$

$$\Phi_2 = \frac{\alpha\beta\left(\Phi_1(1-\delta) + y/p\right)}{(1+\beta+\alpha\beta)} \tag{2}$$

Closed form solution for s:

$$s = \frac{p\beta\left(\Phi_1(1-\delta) + y/p\right)}{(1+\beta+\alpha\beta)} \tag{3}$$

Discussion closed form solution (1)

- Higher discount factor, higher income and higher initial stock of literacy are associated with higher saving s.
- s is inversely related with the return to literacy α.
- Notably, $s = \alpha^{-1} p \Phi_2$ (cf. Eqs. (2) and (3)).
  This has several implications:
  1. Incentive to invest in financial literacy depends on α and s.
  2. In a cross-section of households reporting information on financial literacy and saving, we should find a positive **association** between the two variables.

Discussion closed form solution (2)

- If $\delta = 1$, the saving rate s/y doesn't depend on income: $s/y = (1 + \beta + \alpha\beta)^{-1}$.
- However, in that case, the wealth-to-income ratio a/y depends on y, as stressed by Lusardi et al. (2017))!

$$a/y = Rs/y = s\phi/y = \frac{\beta^2 y}{(1+2\beta)^2 p} \tag{4}$$

- Assuming a fixed R and thus no investment in knowledge, the solution for wealth is proportional to income:

$$a/y = R\beta/(1+\beta)$$

- So, investment in financial knowledge might explain why the highly educated have a higher wealth-to income ratio.
- In a model with endogenous knowledge, there is a complementarity between an agent's need to save and his willingness to invest to raise R.

A stylized multiperiod model (T periods), Japelli et al.

Value function

$$V_t(A_t, \Phi_t) = \frac{c_\tau^{1-\gamma}}{1-\gamma} + \beta V_{t+1}(A_{t+1}, \Phi_{t+1})$$

where

$$A_{\tau+1} = \Phi_{\tau+1}^{\alpha} \left(A_\tau + y_\tau - c_\tau - p(\Phi_{\tau+1} - (1-\delta)\Phi_\tau)\right)$$

Euler equation

$$\frac{c_{t+1}}{c_t} = \beta \left(\Phi_{t+1}^{\alpha}\right)^{1/\gamma}$$

Φ_{t+1} evolves according to the following recursion

$$\left(p - \alpha \frac{s_t}{\Phi_{t+1}}\right) \Phi_{t+1}^{\alpha} - p(1-\delta) = 0, \quad \text{for } t \leq T-2 \qquad (5a)$$

$$p - \alpha \frac{s_t}{\Phi_{t+1}} = 0, \quad \text{for } t = T-1 \qquad (5b)$$

where $s_t = A_t + y_t - c_t - p\left(\Phi_{t+1} - (1-\delta)\Phi_t\right)$.

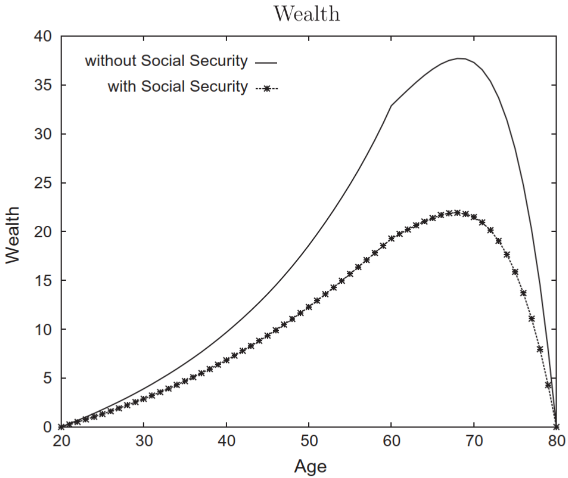
- Euler equation: consumption growth is positively related to the interest rate, and therefore increases with the return to literacy (α) and the stock of financial literacy Φ_{t+1}.
- The sequence of optimal c_t and Φ_{t+1} can be found by solving the system given by the Euler equation, the budget constraint, and eqs. (5).

Illustration multiperiod model

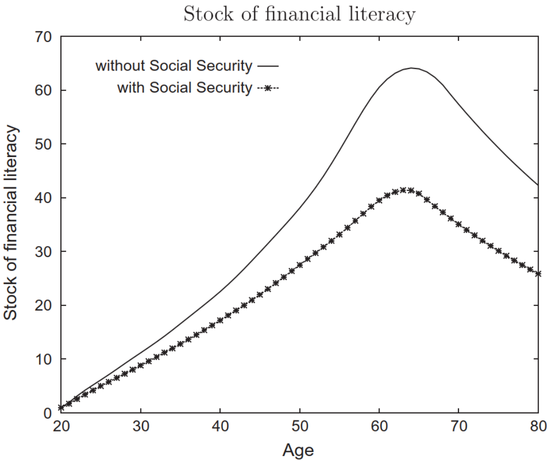
- Assumptions

  1. $T = 60$;
  2. **Scenario 1:** In the first 40 periods constant income; in the last 20 periods (retirement), only interest income.
  3. **Scenario 2:** During working life, consumer contributes 20% of income to a social security system. During retirement, an actuarially fair and constant pension.
  4. The other parameters are the following: $\alpha = 0.03$, $\beta = 0.99$, $\delta = 0.03$, $p = 0.1$, $\Phi_1 = 1$, $\gamma = 2$ and first-period income equals 1.

Age profile of wealth (A_t) in the multiperiod model



Age profile of the stock of financial literacy (Φ_t)



Comments on figures 21 and 32 (1)

- Age profile of wealth hump shaped (standard result).
- Stock of literacy has a similar age-profile, increasing in the first portion of the life-cycle and decreasing after retirement.
- During retirement, consumers still purchase financial literacy ($\phi_t > 0$) but stock of financial literacy (Φ_t) falls because the effect of depreciation dominates.
- Only in the final period, there is no incentive to invest in Φ_t.

Comments on figures 21 and 32 (2)

- Social security system reduces not only the incentive to accumulate assets, but also investment in financial literacy.
- The reduction in wealth accumulation (top panel) is the familiar displacement effect induced by social security.
- Since the incentive to invest in financial literacy depends on the amount saved, with social security consumers also accumulate a lower stock of financial literacy.
- "Although we do not model explicitly the composition of household portfolios, this might explain why in countries with more generous social security systems households participate less in financial markets and have relatively simpler portfolios."

Features of the model of Lusardi et al. (2017)

- It allows for uncertainty in income, capital market returns, and medical expenditures.
- It incorporates an endogenous knowledge accumulation process and a sophisticated saving technology.
- Financial knowledge permits consumers to use sophisticated financial products that can help them raise the return earned on saving.
- Individuals who wish to transfer resources over time by saving will benefit most from financial knowledge.
- Moreover, because of how the US social insurance system works (along with many other such systems around the world), better-educated individuals have the most to gain from investing in financial knowledge. As a result, making financial knowledge accumulation endogenous allows for an amplification of differences in accumulated retirement wealth over the life cycle.

The model of Lusardi et al. (2017)

Lusardi et al. (2017) extend the simple LCH model in several directions:

- **Uncertainty** in a) asset returns, b) household income y_t and c) out-of-pocket (oop_t) medical expenditures plus mortality risk.

  Heterogeneity in uncertainty across 3 educational groups ($<$ HS, HS, College+).

- **Preferences**:
  - $n_t u(c_t/n_t)$ ($n_t :=$ education specific equivalence scale).
  - n_t is hump-shaped over the life cycle which contributes to generate a hump-shaped consumption profile.
  - $u(.)$ CRRA with $\sigma = 1.6$ (no heterogeneity in preferences)

- Consumer is eligible for a government transfer $tr_t = \max(c_{min} - x_t, 0)$. Cash on hand x_t is equal to

$$x_t = a_t + y_t - oop_t$$

The model of Lusardi et al. (2017) (2)

- Consumer can choose between 2 different investment technologies
  1. Basic technology with return $\bar{r}$ ($\bar{R} = 1 + \bar{r}$).
     $\bar{r}$ represents the return to consumers without any financial know-how.
  2. Sophisticated technology with a higher expected return, which increases in financial knowledge f_t but comes at a cost.
  3. Cost structure sophisticated technology
     - c_d:=Fixed costs (fee) c_d
     - $\pi(i_t)$:= Variable costs (in the form of time or money)) in acquiring one extra unit financial knowledge i_t
       $\pi(i_t)$ convex cost function (same for everybody)
     - Note: sophisticated technology cannot be purchased if $x_t - c_d < c_{min}$ (i.e., the government will not pay for costs of obtaining the technology)
- $f_t = (1 - \delta) f_{t-1} + i_t$
- The rate of return on the sophisticated technology is stochastic

$$\ln(\tilde{R}(f_t)) = \bar{r} + r(f_t) + \sigma_\varepsilon \varepsilon_t$$

The model of Lusardi et al. (2017) (3)

- End-of-period assets are given by

$$a_{t+1} = R(f_{t+1})(x_t + tr_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0))$$

where $R(f_{t+1}) = \kappa_t \tilde{R}(f_t) + (1 - \kappa_t)\bar{R}_t$.

κ_t= fraction of wealth invested in the sophisticated technology.

Income process

$$\log y_{e,t} = g_{y,e}(t) + \mu_{y,t} + \nu_{y,t},$$
$$\mu_{y,t} = \rho_{y,e}\mu_{y,t-1} + \varepsilon_{y,t},$$
$$\varepsilon_{y,t} \sim N\left(0, \sigma^2_{y,\varepsilon}\right), \quad \nu_{y,t} \sim N\left(0, \sigma^2_{y,v}\right).$$

- Postretirement income is a function of preretirement income
- Education specific replacement rates: 0.75 for $< HS$; 0.74 for HS, and 0.63 for $college+$

Process oop medical expenditures

$$\log\ \text{oop}_{e,t} = g_{o,e}(t) + \mu_{o,t} + \nu_{o,t},$$

$$\mu_{o,t} = \rho_{o,e}\mu_{o,t-1} + \varepsilon_{o,t},$$

$$\varepsilon_{o,t} \sim N\left(0, \sigma^2_{o,\varepsilon}\right), \quad \nu_{o,t} \sim N\left(0, \sigma^2_{o,\upsilon}\right).$$

Value function

State vector $s_t = (\mu_{y,t}, \mu_{o,t}, e, f_t, a_t)$

$$
\begin{aligned}
V_d(s_t) &= \max_{c_t, i_t, \kappa_t} n_{e,t} u(c_t / n_{e,t}) \\
&\quad + \beta p_{e,t} \int_{\varepsilon} \int_{\eta_y} \int_{\eta_o} V(s_{t+1}) dF_e(\eta_o) dF_e(\eta_y) dF(\varepsilon), \\
a_{t+1} &= \tilde{R}_{\kappa}(f_{t+1})[a_t + y_{e,t} + \text{oop}_{e,t} + tr_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0)], \\
f_{t+1} &= (1 - \delta) f_t + i_t, \\
\tilde{R}_{\kappa}(f_{t+1}) &= (1 - \kappa_t)\bar{R} + \kappa_t \tilde{R}(f_t),
\end{aligned}
$$

Calibration

TABLE A1
BASELINE PARAMETER VALUES

Parameter	Definition	Value
σ	Relative risk aversion	1.6
β	Discount factor	.96
$\bar{r}$	Safe return	.02
σ_e	Standard deviation returns	.16
$r(f_{\max})$	Maximum excess return	.04
π_0	Productivity of investment function $(\pi(i_t) = \pi_0 i_t^{\pi_1})$	50
π_1	Concavity of investment function	1.75
c_d	Participation cost	750
δ	Depreciation rate	.06
$c_{\min}$	Consumption floor	10,000
ρ_y	Autocorrelation income	.95
$\sigma^2_{y,\varepsilon}$	Variance innovation income (<HS, HS, college+)	(.033, .025, .016)
ρ_o	Autocorrelation out-of-pocket	.901
$\sigma^2_{o,\varepsilon}$	Variance innovation out-of-pocket (<HS, HS, college+)	(.175, .156, .153)

Simulated and observed outcomes at retirement (age 65)

TABLE 2

SIMULATED AND OBSERVED OUTCOMES AT RETIREMENT (Age 65)

	< High School	High School	College+	Ratio (College+/ < High School)
	Simulation			
Median wealth	94,746	177,391	346,805	3.66
Average income	31,780	38,703	47,485	1.494
Wealth-to-income ratio	2.981	4.583	7.303	2.45
Fraction poor ($a_t > 2y_t$)	.387	.2903	.1742	.4501
Participation ($\kappa_t > 0$)	.4502	.6123	.7811	1.735
Share $E[\kappa_t \| \kappa_t > 0]$	.9712	.9748	.9687	.9975
Low FK ($f_t < 25$)	.5382	.3665	.2092	.3888
	Data (PSID)			
Median wealth	101,872	193,392	365,392	3.587
Wealth-to-income ratio	3.21	4.99	7.695	2.40
Fraction poor ($a_t < 2y_t$)	.3478	.3039	.1588	.4565
Participation ($\kappa_t > 0$)	.2817	.6289	.7538	2.676
Share $E[\kappa_t \| \kappa_t > 0]$	.5834	.5231	.5787	.9919

NOTE.—This table summarizes outcomes from baseline simulations at age 65 compared to actual observed outcomes in the PSID. The last column shows the ratio of college+ to

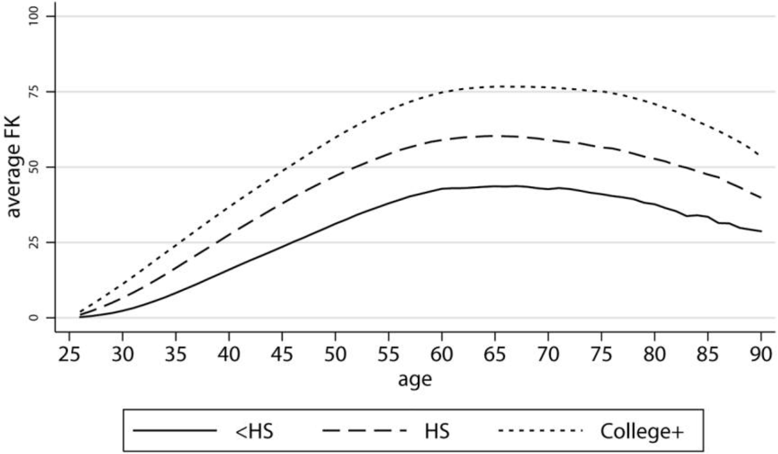
Comments on table 33

- wealth to income ratio increases with education level
- Fraction poor (relative definition of poverty) decreses with education level
- Stock market participation increases with education level.

  One does not obtain this result if one ignores the role of financial knowledge.
- The model does a bad job in simulating $E(\kappa_t|\kappa_t > 0)$.
- The lowly education build up less FK.

  Given the production function for knowledge, a threshold of 25 units implies that such households could expect an annual excess return of only 1 percentage point or less.

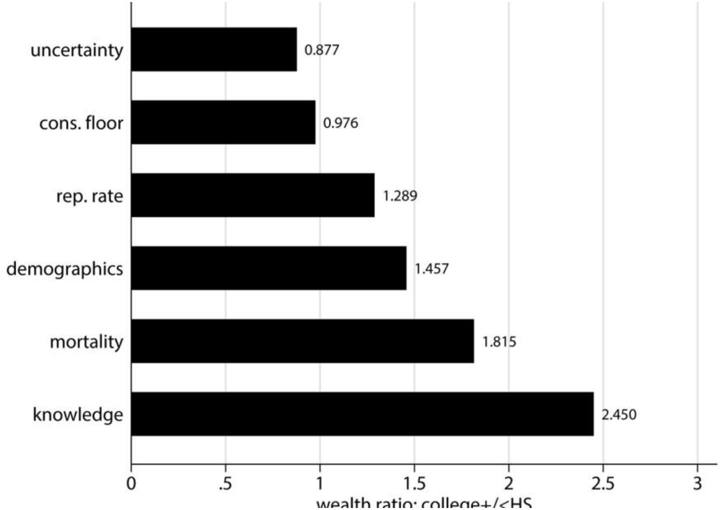
Decomposition of wealth inequality at retirement.



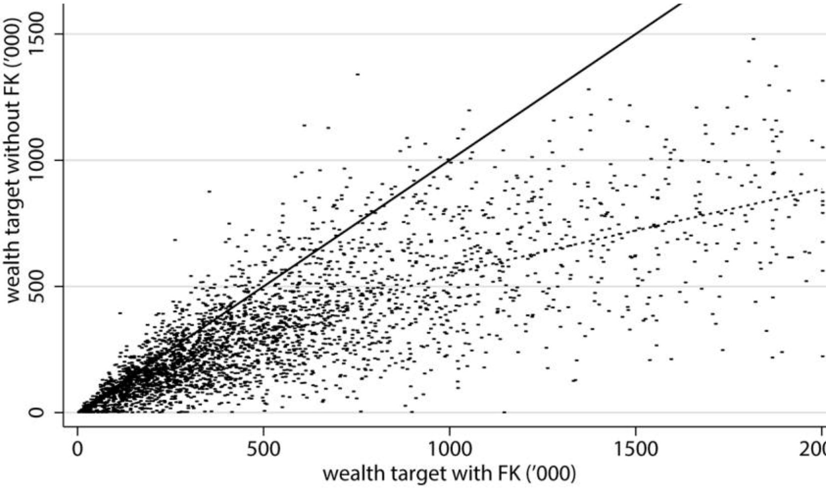
Decomposition of wealth inequality at retirement.

- The model embodies several differences across education groups that generate differential wealth accumulation patterns.
  1. The consumption floor acts as a tax on saving for those most likely to experience a substantial negative income shock, since subsistence benefits are means-tested.
  2. Differences in replacement rates, demographics, and mortality patterns can create differential incentives to save.
  3. Financial knowledge, which creates a positive relationship between normalized wealth and income.

Decomposition of wealth inequality at retirement.



Predicted wealth at retirement: base case and scenario without FK



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